

# THE VOLUME MASS TRANSFER COEFFICIENT IN A BUBBLE BED COLUMN

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Relation for the mass transfer coefficient  $k_L$  and volume mass transfer coefficient  $k_L a$  in bubble beds has been derived and its validity confirmed with published experimental results.

The derivation of the expression for the volume mass transfer coefficient  $k_L a$  begins with two basic assumptions:

1. The mechanism of mass transfer can be described by the penetration theory; no interfacial turbulence occurs on contact of the gas with the liquid.

2. To the liquid, which is the continuous phase in bubble bed columns, the concept of isotropic turbulence may be applied whole for the contact time,  $t$ , appearing in the expression for the mass transfer coefficient following from the penetration theory in the form

$$k_L = (2/\sqrt{\pi}) (D_L/t)^{1/2} \quad (1)$$

one can use Kolmogorov's formulations<sup>1-3</sup>.

For the partial coefficient of mass transfer we can then derive the following relation

$$k_L \sim \frac{2}{\sqrt{\pi}} \left[ D_L \left( \frac{e_L}{\nu_L} \right)^{1/2} \right]^{1/2} \quad (2)$$

The rate of energy dissipation in the liquid<sup>4</sup> related to a unit mass of liquid may be expressed as

$$e_L = \nu g \frac{\mu_g}{\mu_L} \left( \frac{1-e}{e} \right) \quad (3)$$

Combining equations (2) and (3) we obtain

$$k_L \sim \frac{2}{\sqrt{\pi}} \left( \frac{D_L}{\nu_L} \right)^{1/2} \left[ \nu g \left( \frac{\rho_G}{\rho_L} \right) \nu_G \left( \frac{1-e}{e} \right) \right]^{1/4} \quad (4)$$

Adopting further the assumption that the porosity of a bubble column does not depend on the flow rate of liquid<sup>5,6</sup> and the dependence on the superficial velocity of gas may be satisfactorily described by<sup>7</sup>

$$e = v/(2v + c) \quad (5)$$

(the value of the constant  $c$  ranges for individual systems roughly between 5 and 50 for  $v$  in  $cm/s$ ), we obtain after some arrangement

$$k_L \sim \frac{2}{\sqrt{\pi}} \left( \frac{D_L}{v_L} \right)^{1/2} \left[ g \left( \frac{\rho_G}{\rho_L} \right) v_G (v + c) \right]^{1/4} \quad (6)$$

The last equation evidences the insignificance of the dependence of the mass transfer coefficient on the superficial velocity of gas which is quite in line with the so far gathered experimental information<sup>7,9</sup> and confirms the premisses of our derivation. In view of the fact that the relation (2) has been derived on the basis of a simplified model concept of the penetration theory (coefficient  $2/\sqrt{\pi}$ ) it would be necessary to take the true empirical coefficient into the relation (2) characterizing the effect of the real system under consideration in order to make the expression (6) an equality.

For the volume coefficient of mass transfer, which is far more easily accessible to experimental measurement than  $k_L$  proper and the knowledge of which is indispensable in a number of practical applications of bubble columns, we need also an expression for the interfacial area. The dependence of the specific interfacial area on the flow rate of gas shall be expressed for this purpose

a) as an empirical power-law expression (*e.g.* in accord with ref.<sup>10</sup>) in the form

$$a \sim c_1 v^{c_2} \quad (7)$$

b) using an expression derived earlier<sup>11</sup> by the author of this communication where it was started practically from the same premisses as those used in this work as derive an expression for  $k_L$ . Namely:

$$a \sim (\rho_L/\sigma)^{0.6} (\mu_g/\mu_L)^{0.4} v^{0.4} (1 - e)^{0.4} e^{0.6} \quad (8a)$$

or by expressing porosity with the aid of Eq. (5) as

$$a \sim (\rho_L/\sigma)^{0.6} (\mu_g/\mu_L)^{0.4} (v + c)^{0.4} v/(2v + c) \quad (8b)$$

Combining expressions (4) and (7) or (4) and (8) we arrive at general correlation formulas for the volume mass transfer coefficients  $k_L a$  in the form

$$k_L a \sim \left(\frac{D_L}{v_L}\right)^{1/2} \left(g \frac{\rho_g}{\rho_L} v_G\right)^{1/4} \left[\left(\frac{1-e}{e}\right)^{1/4} v^{(1/4+c_2)}\right] \quad (9)$$

or

$$k_L a \sim D_L^{1/2} \frac{g^{1/4}}{\sigma^{3/5}} \frac{v_G^{13/20}}{v_L^{9/10}} \frac{\rho_G^{13/20}}{\rho_L^{1/20}} [v^{13/20} e^{7/20} (1-e)^{13/20}]. \quad (10)$$

Substituting for the porosity we obtain

$$k_L a \sim \left(\frac{D_L}{v_L}\right)^{1/2} \left[g \frac{\rho_G}{\rho_L} v_G\right]^{1/4} (v+c)^{1/4} v^{c_2} \quad (11)$$

or

$$k_L a \sim D_L^{1/2} \frac{g^{1/4}}{\sigma^{3/5}} \frac{v_G^{13/20}}{v_L^{9/10}} \frac{\rho_G^{13/20}}{\rho_L^{1/20}} \left[(v+c)^{13/20} \cdot \frac{v}{2v+c}\right]. \quad (12)$$

The course of the functional relationship

$$(v+c)^{13/20} v/(2v+c) \quad (13)$$

appearing in the proportionality (12) may be approximated over a rather wide interval of gas velocities between 0 and 25 cm/s by a power expression  $v^x$ ; for the constant  $c = 20$  cm/s, which is thought to be universal for water solutions of electrolytes<sup>6</sup> and which for this purpose will be regarded as a mean we obtain for the optimum theoretical value  $x = 0.82$ .

This theoretical conclusion is in remarkable agreement with the experimentally found values of  $x$  in the relation

$$k_L a \sim v^x \quad (14)$$

as these were published recently for various solutions of electrolytes and air by Deckwer and coworkers<sup>10</sup>:  $x(0.225N-H_2SO_4) = 0.825$ ;  $x(0.17M-NaCl) = 0.884$ ;  $x(H_2O) = 0.884$ . The gas distributor used in this work was of the orifice sparger type. In reality various values of  $c$  for different electrolytes in Eq. (13) affect the value of the exponent  $x$ , a fact which may account for the different empirical results of  $x$  given by Deckwer\*. In region of low gas velocities (0–3 cm/s) the functional relationship (13) may be approximated by a power expression (14) with the exponent equalling

\* Actually various combinations of the values of  $c$  and  $v$  yield the theoretical values of the exponent between 0.7 and 1.2.

1.0. Our experimental data confirm this finding (Fig. 1) and the values of the exponent  $x$  exceeding unity in region of small gas velocities were also observed by other authors<sup>12</sup>.

The proportionalities (10) or (12) may be arranged by introducing Reith's relation for the porosity, Eq. (5), to obtain after some simplification

$$k_L a \sim e \left[ c \left( \frac{1-e}{1-2e} \right)^{0.65} \right]. \quad (15)$$

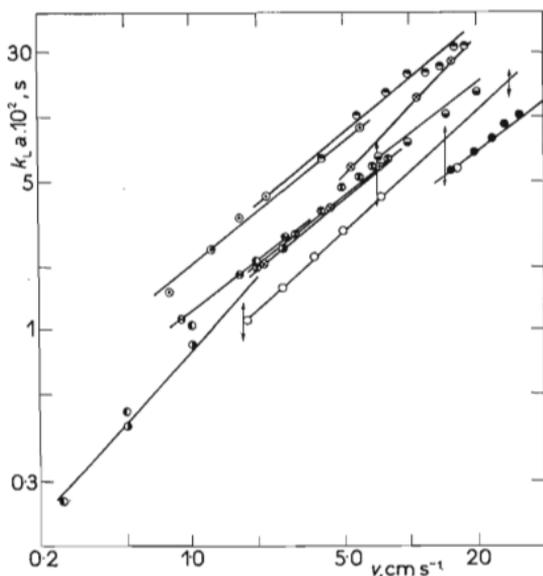


Fig. 1

#### Volume Mass Transfer Coefficient as a Function Gas Velocity

Arrows indicate scatter of experimental points given by Towell<sup>13</sup> (desorption of  $\text{CO}_2$  from water, column diameter 406 mm); ● own data, column diameter 300 mm; ● own data, column diameter 1000 mm, bed height 1200 mm, perforated plate  $\varnothing$  0.5%,  $d_0$  1.6 mm (absorption of  $\text{CO}_2$  from air in water stripped of oxygen by adding  $\text{Na}_2\text{SO}_3$  and  $\text{Co}^{2+}$ ); ⊗ Reith<sup>7</sup> column diameter 290 mm, ⊖ Reith<sup>7</sup> column diameter 140 mm (oxidation of  $\text{Na}_2\text{SO}_3 + \text{Co}^{2+}$  by oxygen); ○ Akita<sup>6</sup> column diameter 150 mm, ( $\text{H}_2\text{O}-\text{O}_2$ ); ⊙ Akita<sup>7</sup> column diameter 150 mm, ( $\text{Na}_2\text{SO}_3-\text{O}_2$ ); ⊕ Deckwer<sup>10</sup> column diameter 200 mm (absorption of  $\text{O}_2$ , 0.225 N- $\text{Na}_2\text{SO}_4$ ); ⊖ Deckwer<sup>10</sup> column diameter 200 mm (absorption of  $\text{O}_2$  -0.17M- $\text{NaCl}$ ); ● Mashelkar<sup>14</sup> column diameter 100 mm (chemisorption  $\text{CO}_2-\text{Na}_2\text{CO}_3/\text{NaHCO}_3$ ); ● own data<sup>15</sup> column diameter 150 mm,  $h$  600 mm, ( $\text{CO}_2$ -water solution of ammonia, chemisorption); ⊕ own data (chemisorption,  $\text{CO}_2-\text{K}_2\text{CO}_3/\text{NaHCO}_3(\text{ClO}^-)$ ), column diameter 150 mm,  $h$  1100 mm.

The last expression may be approximated for  $v$  ranging between 1 and 20 cm/s and  $c = 20$  by the following function relationship with the maximum deviation of 12%

$$k_L a \sim v^y. \quad (16)$$

For the optimum value of the exponent it was found  $y = 1.16$ . Akita and Yoshida<sup>6</sup> found experimentally by measuring a series of aqueous as well as non-aqueous solutions the value  $y = 1.1$ .

The agreement of both values is remarkable (the empirical correlation of Akita and our correlation were found also to agree in the exponents over the diffusion coefficient and the surface tension).

The approximation of the functional relation (13) by a power expression follows, of course, very clearly from (11) where in view of the absolute values of  $v$  and  $c$  and the exponent of  $1/4$  the power law dependence between  $k_L a$  and  $v^{c_2}$  is evident.

Fig. 1 shows selected experimental values of  $k_L a$  of a number of authors<sup>6,7,10,13,14</sup> including the author's of this communication in dependence on the velocity of gas. All experimental points appear in the log-log coordinates on straight lines with their slopes ranging between 0.78 and 1.12; the mean slope of these data is  $c_2 = 0.84$

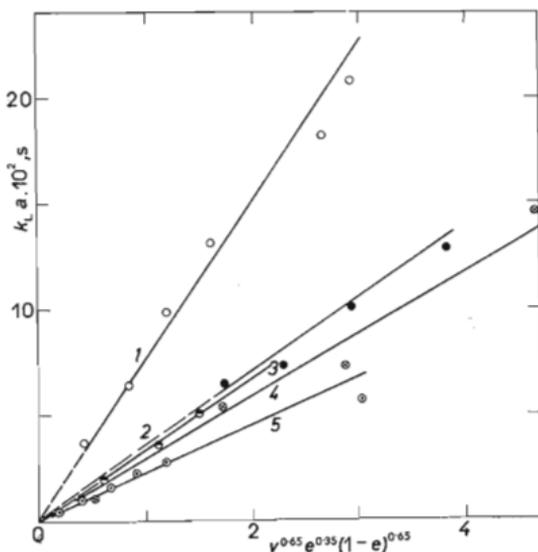


FIG. 2

Volume Mass Transfer Coefficient as Given by Correlation (10)

1 Our data<sup>15</sup>, 2 Reith<sup>7</sup>, 3 Deckwer<sup>10</sup>, 4 Towell<sup>13</sup>, 5 Akita<sup>6</sup>,  $v$  in  $\text{cm s}^{-1}$ .

which is in very good agreement with our theoretical prediction. (The geometrical parameters of the columns range as follows: height 60 to 723 cm, diameter 10 to 100 cm).

Fig. 2 is a plot of the same experimental data according to the general correlation (10). Excepting the data of Akita, which somewhat deviate at higher gas velocities from the straight course, the available experimental evidence confirms our correlation. (Only data of those authors could be taken who measured also the mean porosity of the bubble bed).

In view of the various column diameters used, height of bubble beds, the various systems and measuring techniques and further in view of the possible objective causes for disagreement<sup>8,15</sup>, the low scatter of the data seems rather surprising. With a maximum error of  $\pm 20\%$  one can estimate the volume mass transfer coefficient from the following correlation

$$k_L a \cdot 10^2 \text{ (s}^{-1}\text{)} = 2.875v^{0.65}e^{0.35}(1 - e)^{0.65}, \quad (17)$$

where  $v$  is in cm/s. The last correlation contains no physico-chemical macroparameters of the gas-liquid system because the data used covered only various solutions of electrolytes whose parameters do not differ markedly enough to produce any significant change of the empirical coefficient in Eq. (17). For non-aqueous solutions there is not enough experimental data on  $k_L a$  available so far to test safely the general correlation (10). Nevertheless, the agreement of the numerical value of the exponent  $y$  in the proportionality (16) suggests even this possibility. Significant deviation display only our data<sup>15</sup> measured with the  $\text{CO}_2\text{-NH}_4\text{OH}$  system. This system, however, exhibits a strong effect of interfacial phenomena which may increase the value of  $k_L$  by a manifold. The dependence (10) though is valid for this case too.

## SYMBOLS

$a$	specific interfacial area relating to bubble bed volumen
$c$	constant characterizing system (Eq. (5))
$d_0$	opening diameter
$D_L$	diffusion coefficient of dissolved gas
$e$	porosity of bubble bed
$e_L$	rate of energy dissipation in a unit mass of liquid mean porosity of bubble bed
$g$	acceleration due to gravity
$h$	height of bed
$k_L$	liquid side mass transfer coefficient
$k_L a$	volume mass transfer coefficient
$t$	contact time
$v$	velocity of gas
$\varphi$	plate free area
$\mu_{G,L}$	viscosity of phases
$\nu_L$	kinematic viscosity
$\rho_{G,L}$	density of phases
$\sigma$	surface tension

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